Models for low frequency oscillations and experiments on driving and external control

A. Smolyakov[1] and Y Raitses[2]
I. Romadanov[1], O. Chapurin[1],
I. Kaganovich[2],
G. Hagelaar[3], S. Sadouni[3], K. Hara[4]

[1]*University of Saskatchewan*
[2]*Princeton University and PPPL*
[3]*LAPLACE & Université Paul Sabatier*
[4]*Texas A&M University*

ExB Plasmas Workshop, PPPL Nov 1-2, 2018

Low-frequency modes: spoke and breathing mode

- Spoke and ion-gradient-drift modes
- Axial modes and ionization effects (breathing mode)
  - Predator-prey model (do not work)
  - Axial resistive modes (important?)
  - Electrothermal (ionization) instability. Likely important.
- Stationary solutions and their stability
  - Stationary state diagram(s)
  - Role of boundary conditions in breathing mode oscillations
- Coupling of axial (breathing mode) and azimuthal (spoke) modes

What do we or do not understand about spoke?

- Azimuthal mode is fundamentally related to the ion-gradient-drift mode and ionization*

$$\omega = k_y L_n \omega_{Bi}$$

$k_y = m/r$ - **azimuthal wave number**

$L_n = -\left(\partial \ln n_0 / \partial x\right)^{-1}$ - **gradient length scale**

$\omega_{Bi} = eB/m_i c$ - **ion cyclotron frequency**

- ion-gradient-drift mode is a former "anti-drift mode" (Friedman, 1964, Timofeev 1963)

$$\omega = \frac{k^2 c_s^2}{\omega_*}$$

$$\omega = \omega_* = -k_y T_e / eBL_n$$

- Gradient-drift is a better name, or even **Ion-Gradient-Drift (IGD)** to distinguish it from the electron drift mode

Disclaimer: The views and **opinions expressed** thereof do not necessarily reflect

… …

Gradient-drift instabilities (Simon-Hoh et al)

- Ion-gradient-drift mode (IGD) is a basis for "collisionless Simon-Hoh instability"

$$\omega^2 = k_y L_n \omega_{Bi} \left(\omega - \omega_E\right)$$

- Original classic Simon, Hoh, 1963 (Timofeev 1963) is collisional

- Collisionless mode is a reactive instability due to the resonance of the IGD mode with degenerate $\omega = 0$ mode shifted by EXB flow . As a result, the instability can be almost aperiodic
- IGD mode can be driven unstable by dissipation, magnetic field gradients, sheath conductivity, ion flow ionization , ...

$$D(\omega, k_y, L_n, L_B, \omega_E, \nu_i, \nu, \nu_s, V_{0i}, ..) = 0$$

$$\nu_s = c_s / L \quad \text{sheath resistivity}$$

- Additional complexities are due to the equilibrium constraints: various plasma parameters/gradients are related to each other

$$F(L_n, L_B, \omega_E, V_{0i}, ..) = 0$$

Axial (breathing modes) oscillations are understood better, but …

- **There are many models for the breathing mode claiming to explain/predict it**
  - **Predator-prey models (ion-neutral coupling)**
  - **Ion-neutral + electric field dynamics+electron conductivity (no diffusion), Morozov, 1995**
  - **…+ electron diffusion and electron energy equation (axial resistive modes**
  - **…electrothermal/temperature dependent ionization effects..**
- **Do we know the right combinations of these effects?**
- **Do we need to include any kinetics effects? e.g. ion temperature/viscosity?**
- Complexity: Interaction of the above effects can result in very stiff behavior—global constraints/profiles

Sonic and global constraints on the ion acceleration and plasma parameters profiles

Approach:

- Find stationary axial profiles
- Analyse their stability
- Analyse the role of boundary conditions on stability

Ion acceleration/ionization/electron current model

$$\frac{\partial n_a}{\partial t} + v_a \frac{\partial n_a}{\partial x} = -\beta n_a n_i$$

Neutrals continuity equation

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_i}{\partial x} = \beta n_a n_i$$

Ions continuity equation

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{e}{m_i} E + \beta n_a (v_a - v_i) \quad (1)$$

Ions momentum equation

$$const = J = n v_i + \mu_e E + D_e \frac{\partial n}{\partial x}$$

Electrons momentum equation and quasineutrality

J_d is the discharge current density, β is the ionisation rate, n_a is neutral density, v_a is neutral velocity (taken as constant), n_i is ion density, v_i is ion velocity.

Assumptions: β, μ_e, T_e are constant along the channel. Recombination on the anode is taken into account.

Stationary solutions. Sonic transitions

Initial conditions are obtained by solving steady-state version of system:

$$v_a \frac{\partial n_a}{\partial x} = -\beta n_a n_i + \nu_w n_i$$

$$\frac{\partial n_i v_i}{\partial x} = \beta n_a n_i - \nu_w n_i$$

$$v_i \frac{\partial v_i}{\partial x} = \frac{e}{m_i} E + \beta n_a (v_a - v_i)$$

$$E = \frac{J_{\mathrm{d}}}{e n_i \mu_e} - \frac{v_i}{\mu_e} - \frac{1}{e n_i} + \frac{\partial T_e n_i}{\partial x}$$

$$n'_i = \frac{1}{c_s^2 - v_i^2}\left(\frac{J \quad - n_i v_i}{\mu_e} + \nu_w n_i v_i + \beta n_a n_i (v_a - 2\beta v_i)\right)$$

$$v'_i = \frac{1}{c_s^2 - v_i^2}\left(v_i \frac{J \quad - n_i v_i}{n_i \mu_e} + \nu_w c_s^2 - \beta n_a \left(v_i v_a + v_i^2 + c_s^2\right)\right)$$

$$\phi'_i = \frac{1}{c_s^2 - v_i^2}\left(v_i^2 \frac{J \quad - n_i v_i}{\mu_e n_i} + \nu_w c_s^2 v_i + \beta n_a c_s^2 (v_a - 2 v_i)\right)$$

where $c_s^2 = T_e/m_i$ - the ion sound velocity, $n_a = J_a - n_i v_i / v_a$, and $J_a = \dot{m}/m_i A$.

Sonic point transition at $v_i = \pm c_s$ requires a special treatment (Barral, Ahedo, Fruchtman, Fisch, Dorf, Raitses, …) Demanding that this point is regular

$$F_1(n, J, J_a) = F_2(n, J, J_a) = F_3(n, J, J_a) = 0$$

This condition is reduced to a relatively simple equation

$$-2n_s^2\left(1 - 2\frac{v_a}{c_s}\right) + ns\left[\frac{\nu_e^*}{\beta}\frac{v_a}{c_s} + 2\frac{J_a}{c_s}\left(1 - 2\frac{v_a}{c_s}\right)\right] - \frac{\nu_e^*}{\beta}\frac{J}{c_s}\frac{v_a}{c_s} = 0.$$

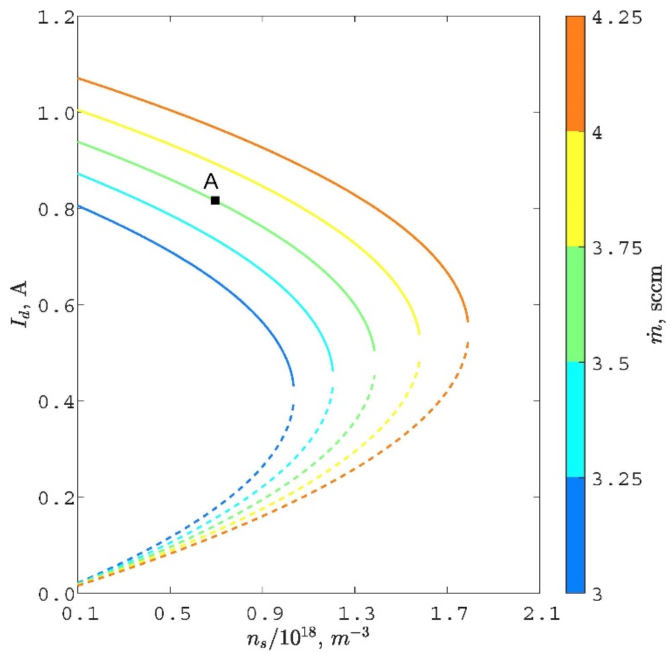
$$\nu_e^\star = ((\omega_{ce}\omega_{ci})/(\nu_e))$$

Quadratic equation relating the plasma density, discharge current and mass injection rate

$$-2n_s^2(1-2\frac{v_a}{c_s})+ns\left[\frac{\nu_e^*}{\beta}\frac{v_a}{c_s}+2\frac{J_a}{c_s}\left(1-2\frac{v_a}{c_s}\right)\right]-\frac{\nu_e^*}{\beta}\frac{J}{c_s}\frac{v_a}{c_s}=0.$$

$$\nu_e^\star=((\omega_{ce}\omega_{ci})/(\nu_e))$$

Stationary solution diagram

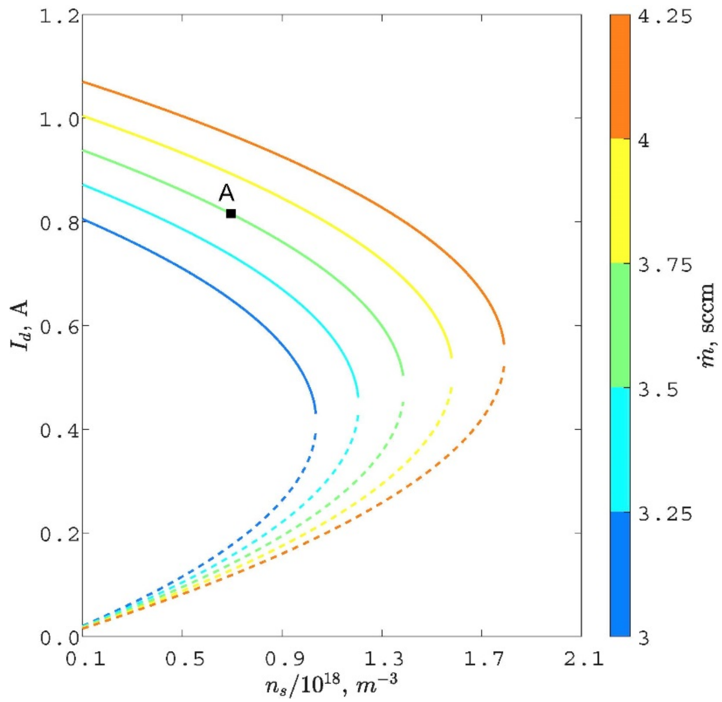


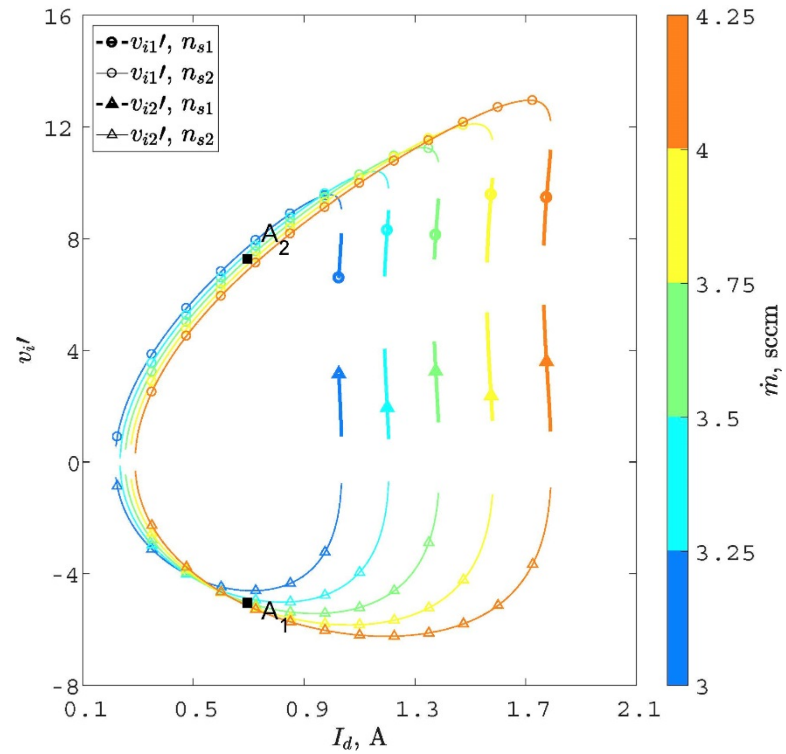
- Two branches: low and high density
- Critical maximal density for a given injection rate

$$n_s=(J,J_a=n_a v_a-nv_i)$$

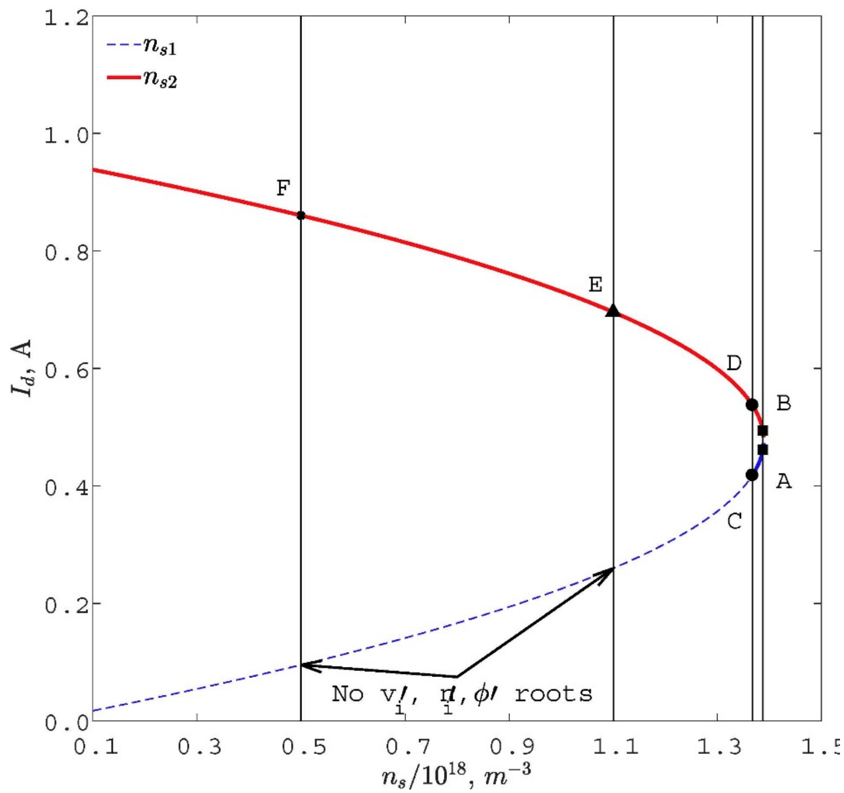
Expanding near the critical point, the equation for the regular values of the velocity gradient can be found

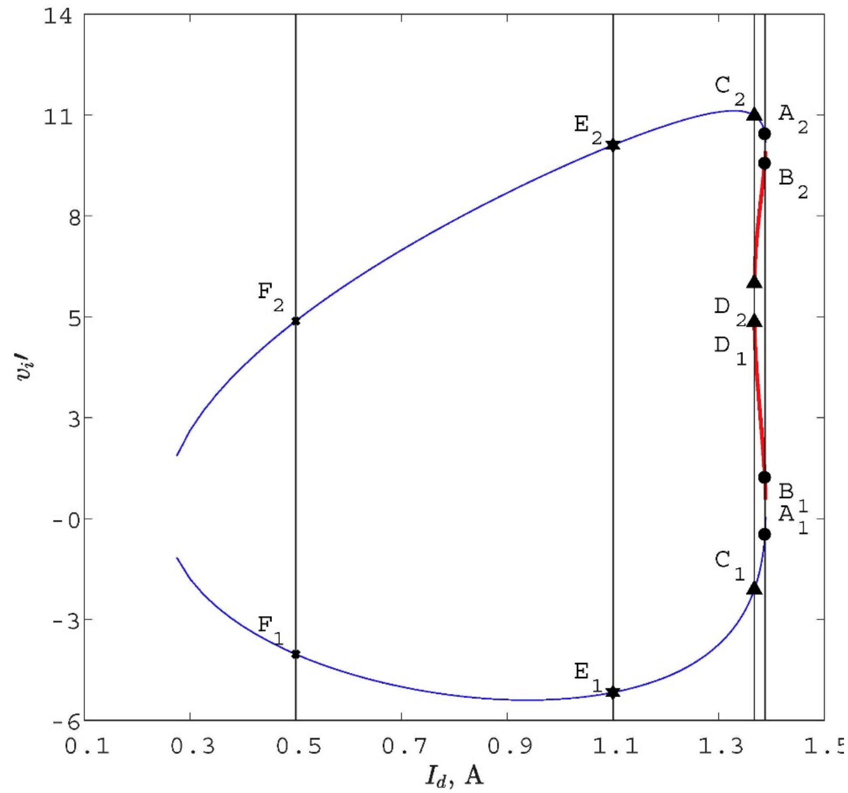
$$v_i'^2 + v_i' \left[\beta n_a \left(1 - \frac{v_a}{2c_s}\right) + \nu_e^* \left(1 - \frac{J}{n_s c_s}\right)\right]$$
$$+\frac{1}{2}\nu_e^* \beta n_a \frac{J}{n_c c_s} + \frac{1}{2}\beta^2 n_a n_s (1 - \frac{2c_s}{v_a}) = 0$$



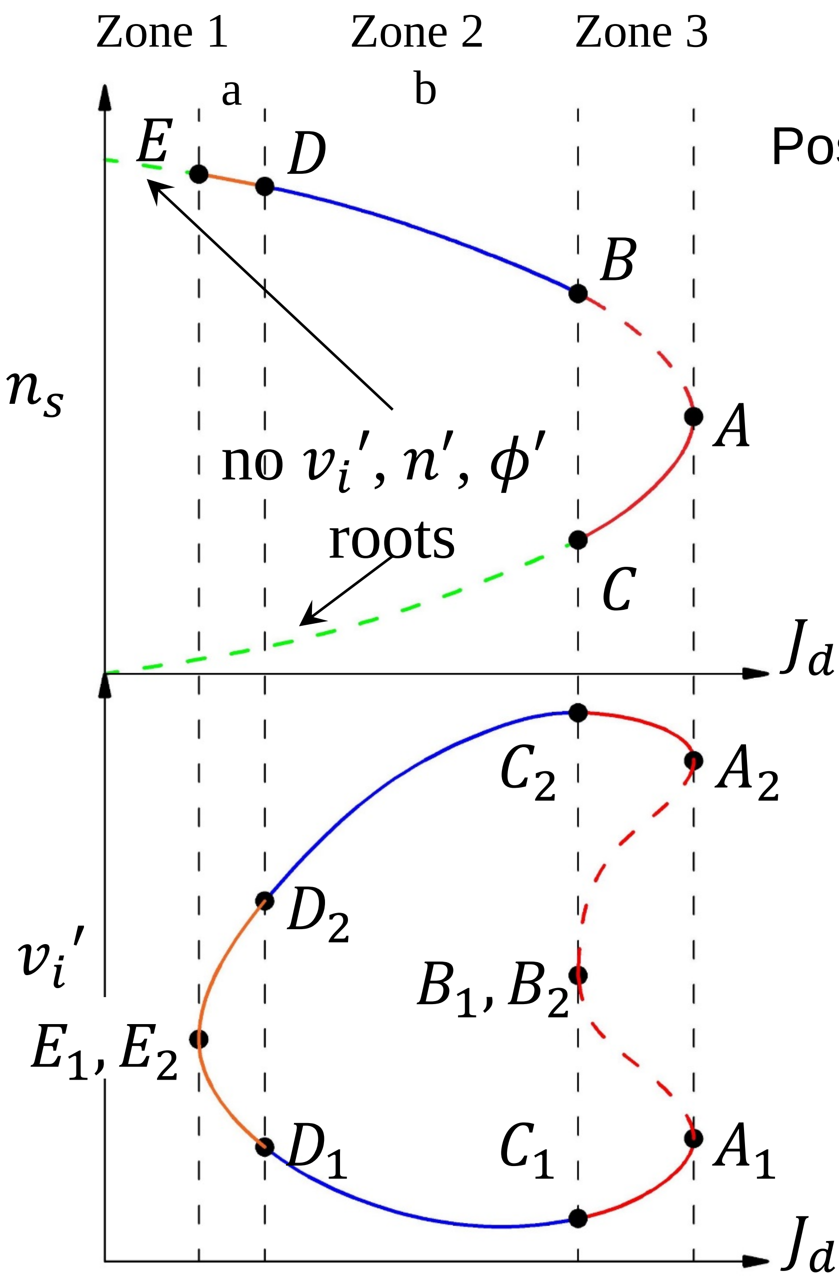


In general: two density roots; each density root generates two values for the velocity derivatives; However some velocity derivatives can be complex; most of the lower density branch does not lead to stationary solution.





Solution diagram in various zones



Positive velocity derivative is on upper branch;

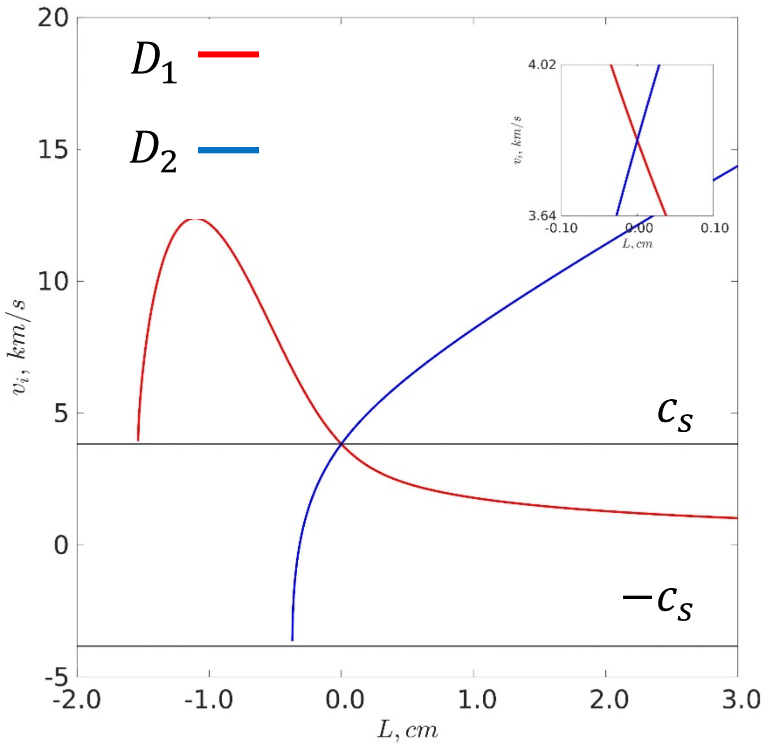
In zone 1 there are no real roots for v_i', n', ϕ'.

In zone 2, only two roots exist for top branch of n_s parabola. In zone 2a, there are no oscillations. In zone 2b single mode oscillations are observed.

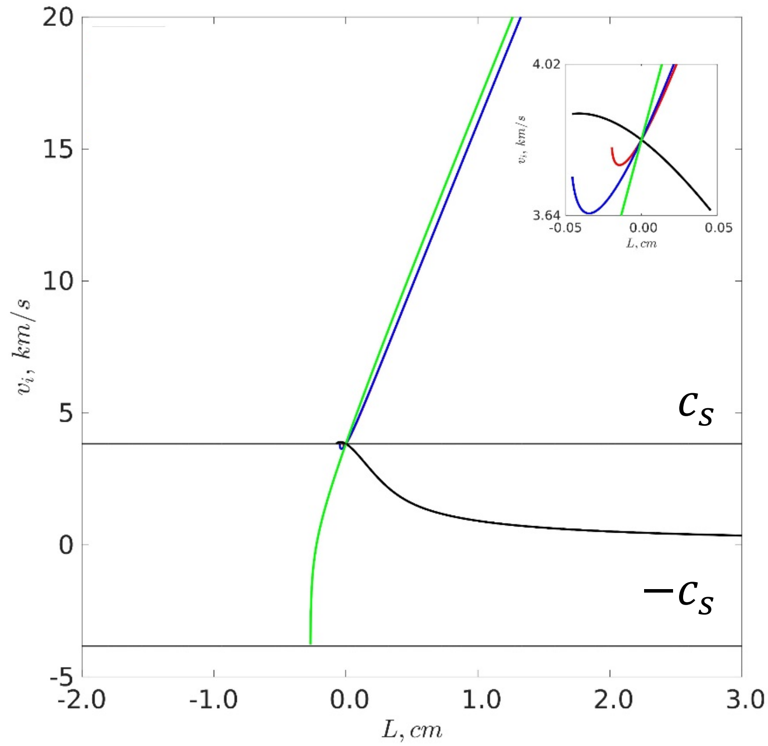
In zone 3, maximum of four roots can exist. Multi-mode oscillations are observed in this zone.

Velocity profile (as well as density,..) are uniquely built

Types of profiles in zone 3



Types of profiles in zone 2b

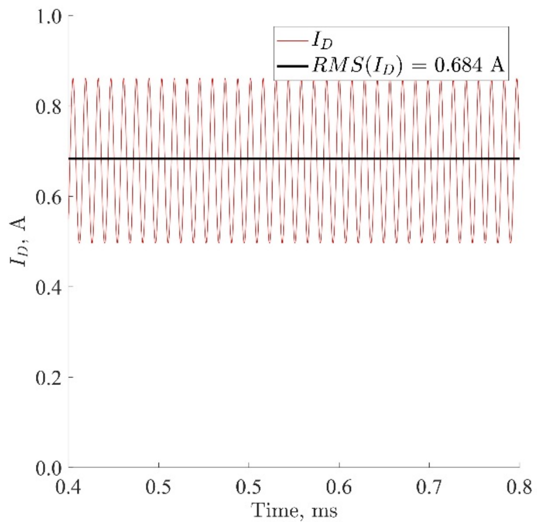


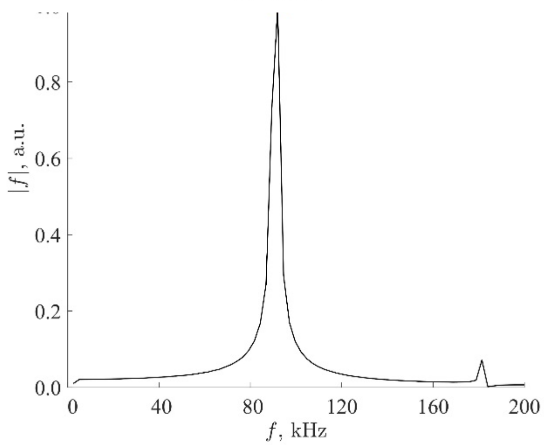
- The whole profiles are built from the sonic point: no freedom for boundary conditions which are fully defined by these global (stiff) solution.
- Then the stationary profiles are analyzed for stability; full time dependent problem is solved starting from the stationary solution as an initial value.
- Stable and unstable regions are identified
- Oscillations occur also when the “wrong” boundary conditions are used for time dependent problem

Character of oscillations is different:
Related to the nonlinear response to external driving?

Zone 2b

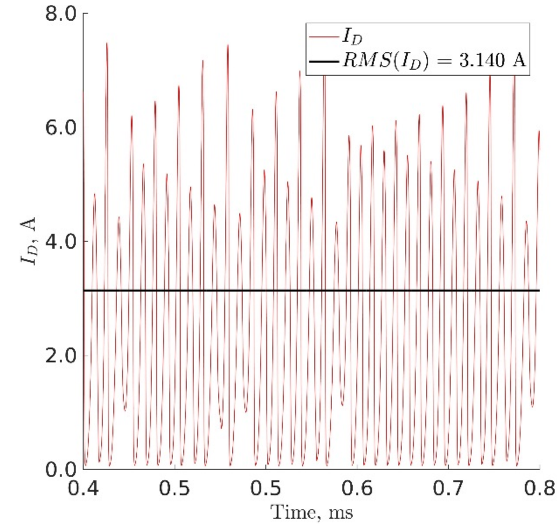
Single mode oscillations

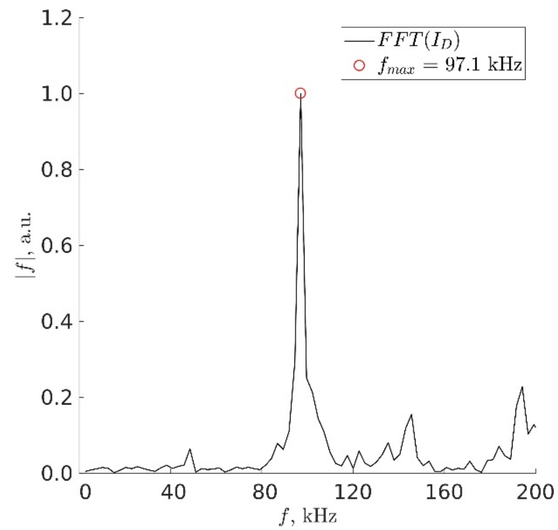




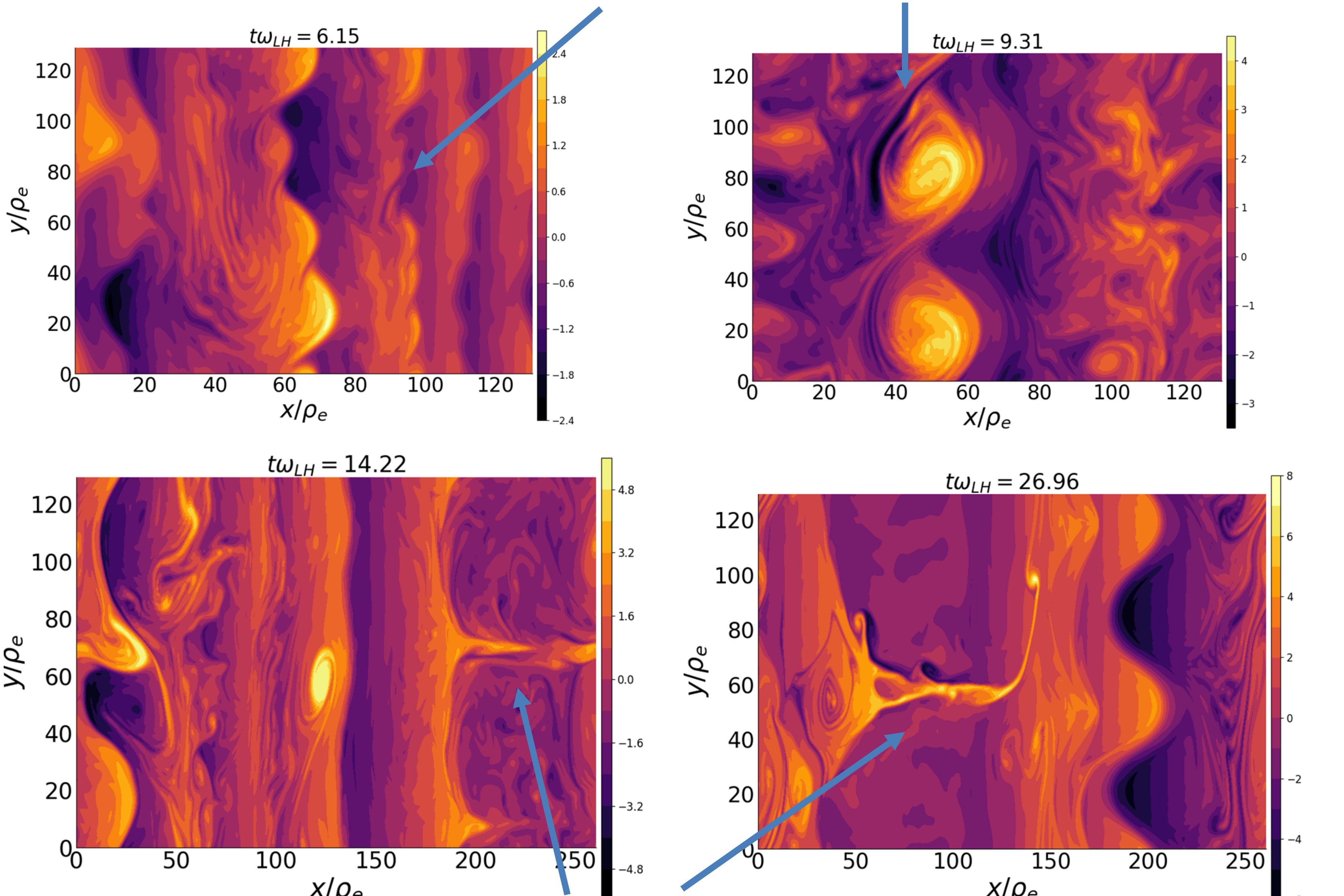
Zone 3

Multimodal oscillations



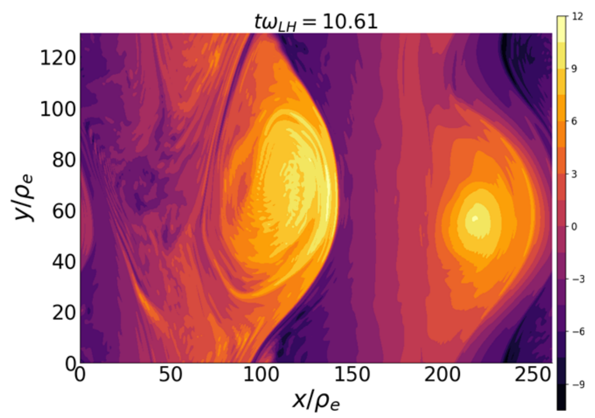


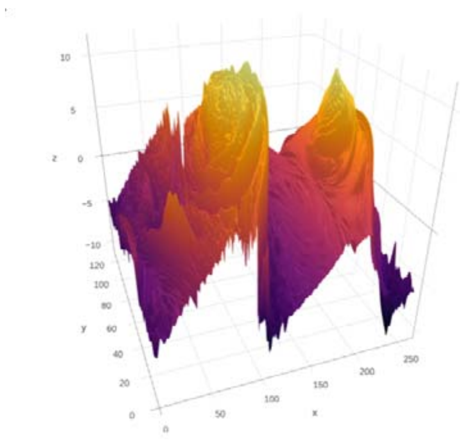
Self-organization: zonal flows, vortices and streamers

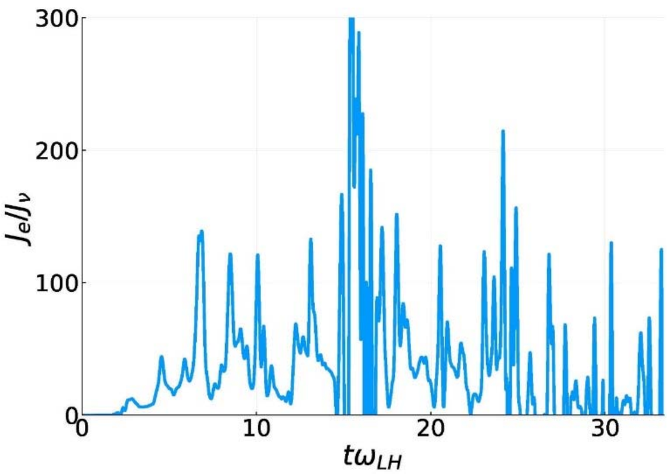


streamers

Coupling of azimuthal and axial modes:



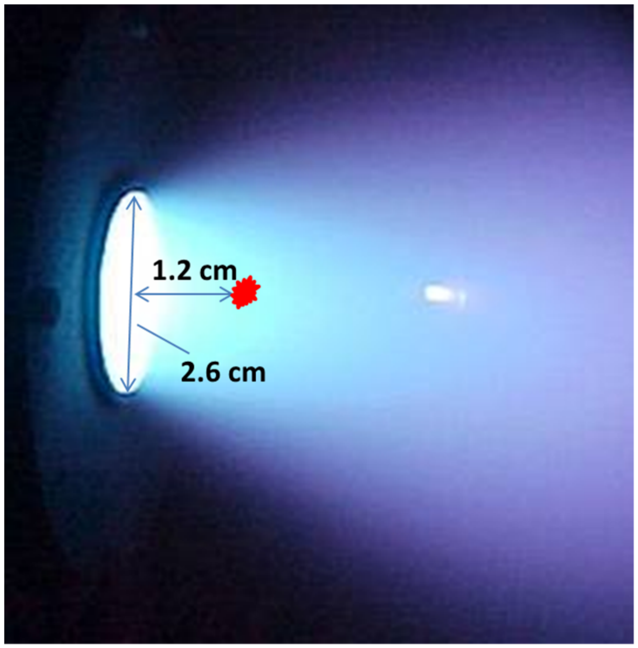


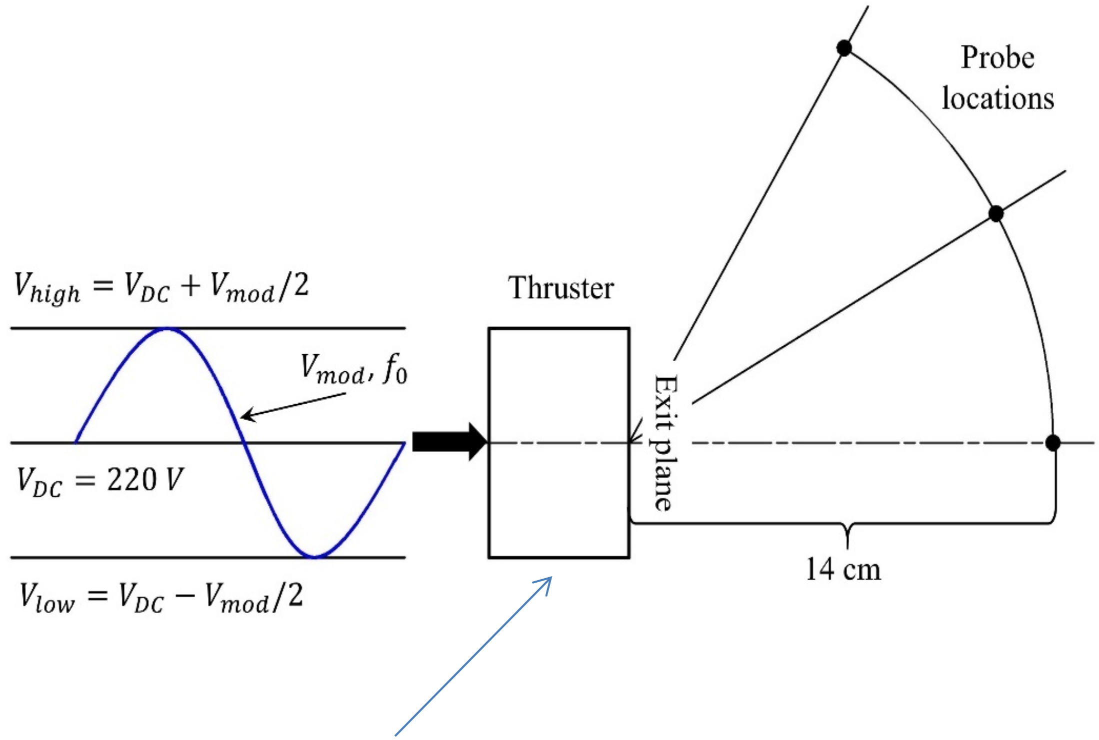


Anomalous current
(associated with azimuthal modes)
driven exclusively by axial modes

Driving of the breathing mode oscillations

- **Modulation of the discharge voltage of the cylindrical Hall thruster**

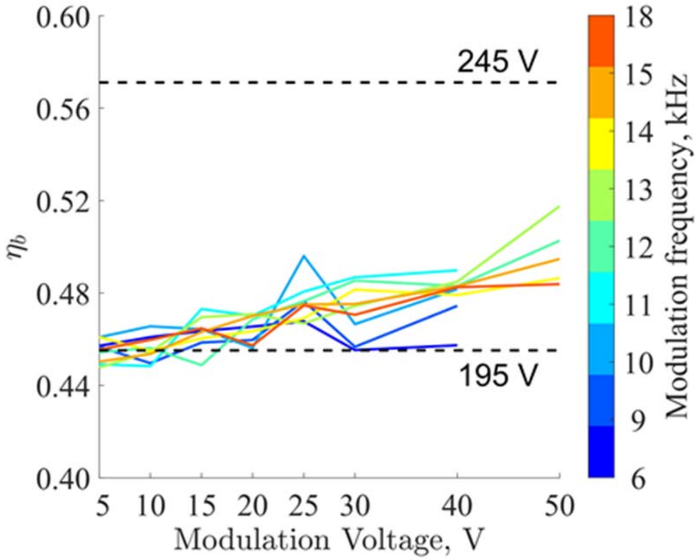




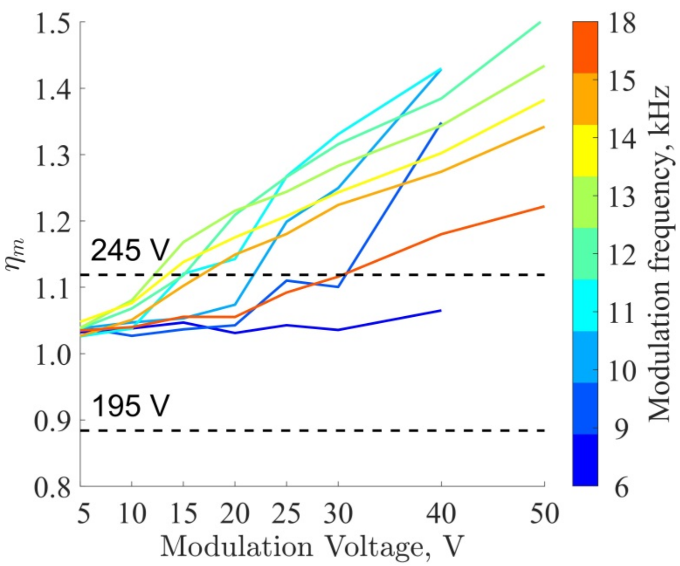
- **Oscillations of the ion current in the plume with an ion saturation probe**
- **IVDF Oscillations by time-resolving Laser-Induced Fluorescence (LIF)**
- **Probes to measure oscillations of plasma properties (plasma potential, electron temperature, plasma density) near the channel exit**

Effect of the driving on ion performance

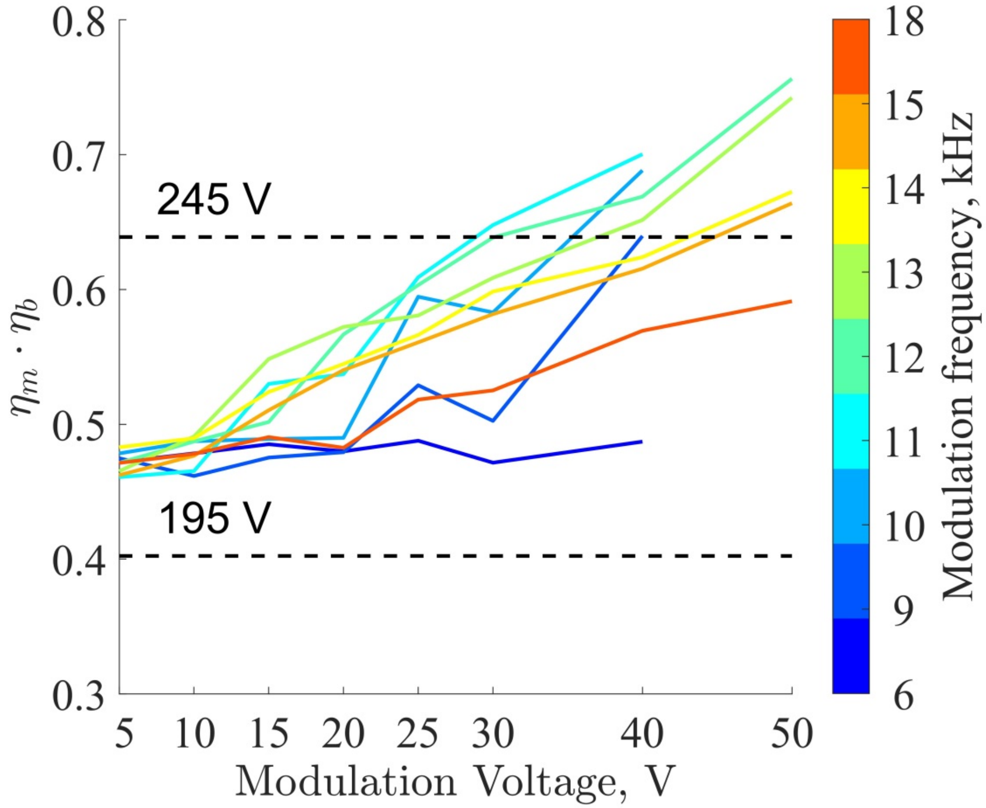
Propellant utilization



Current utilization

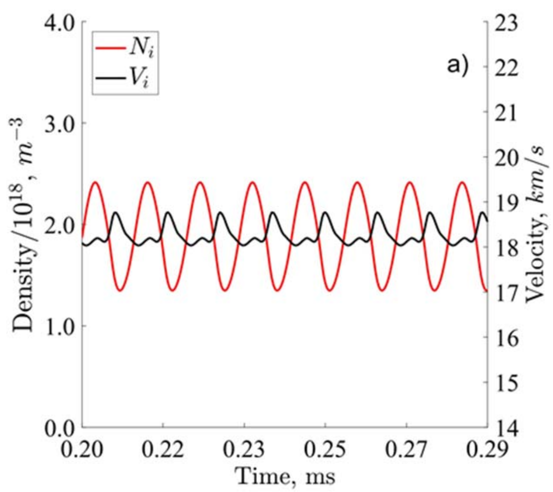


- **Beam efficiency**

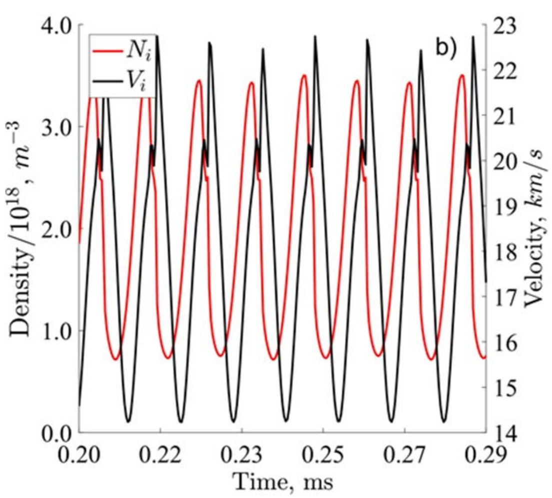


Modeling explained driving effect on plasma properties

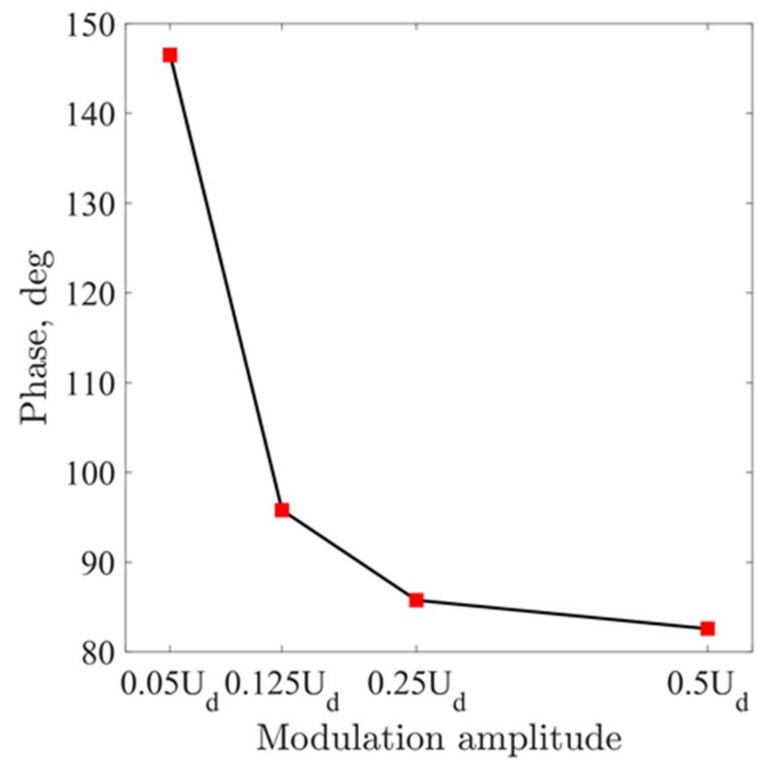
- **1-D Model solved using BOUT++**
  - **Linear mode**



  - **Non-linear mode**



- **Phase difference between ion velocity and ion density**

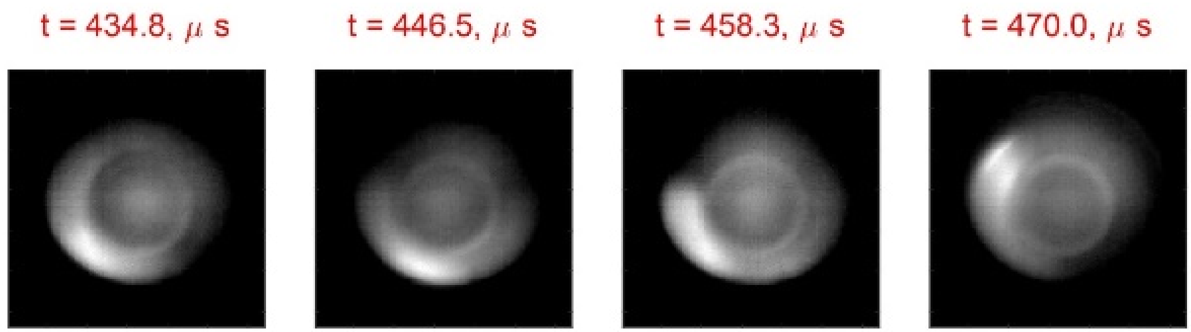


- **Model: modulations reduce the phase, $\Delta\psi$, between oscillations of the ion density N_i and the ion velocity V_i increasing the effective ion flux, J_i, from the thruster:**

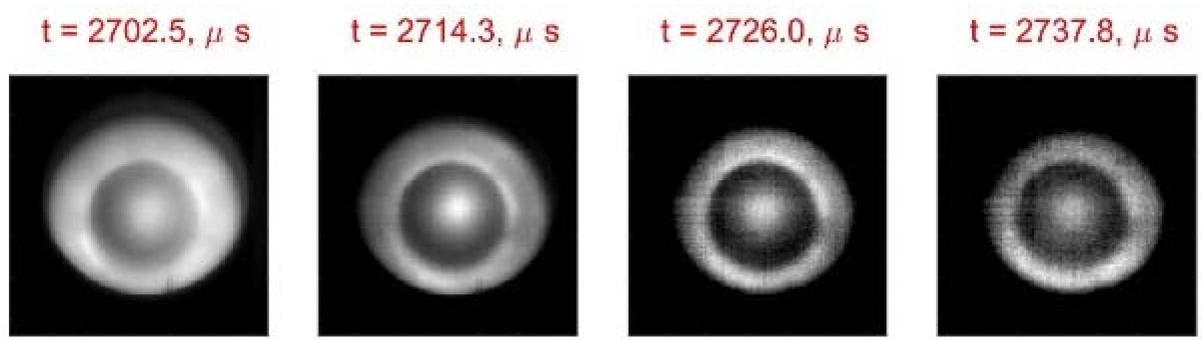
$$J_i \sim N_i V_i \cos(\Delta\psi),$$

Spoke mitigation via coupling with the breathing mode

- **Spoke observed in CHT operation without driving breathing oscillations**



- **Spoke mitigated in CHT at high modulations of breathing oscillations**

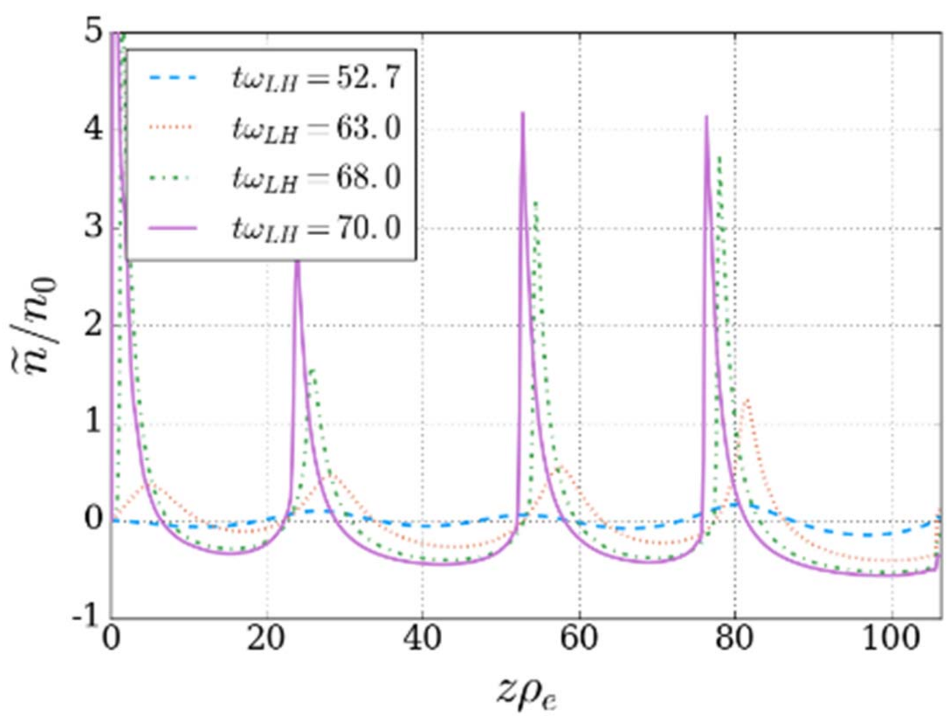


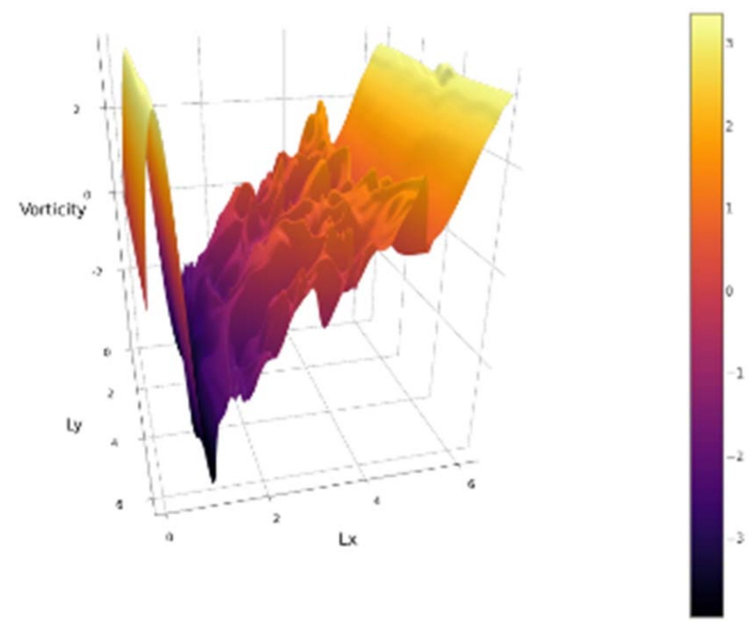
If spoke is an azimuthally propagating mode driven by axial gradients (N_i, T_e), axial gradients (profiles) are modified by driven breathing mode!

Thank you!

Axial-flow instability and large scale structures

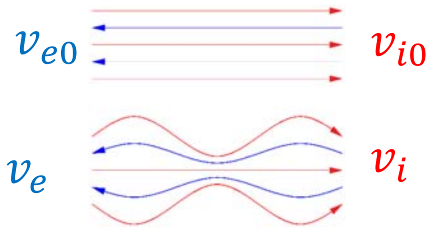
- Axial mode are also found in nonlinear 2D simulations, coexist with small scales (lower hybrid modes)
- Weakly growing structures, hard to saturate, large amplitude, slowly moving, resemble non-monotonous structures in the electric field (Vaudolon, Khiar, Mazouffre 2014)
- Important for ionization (breathing) modes?





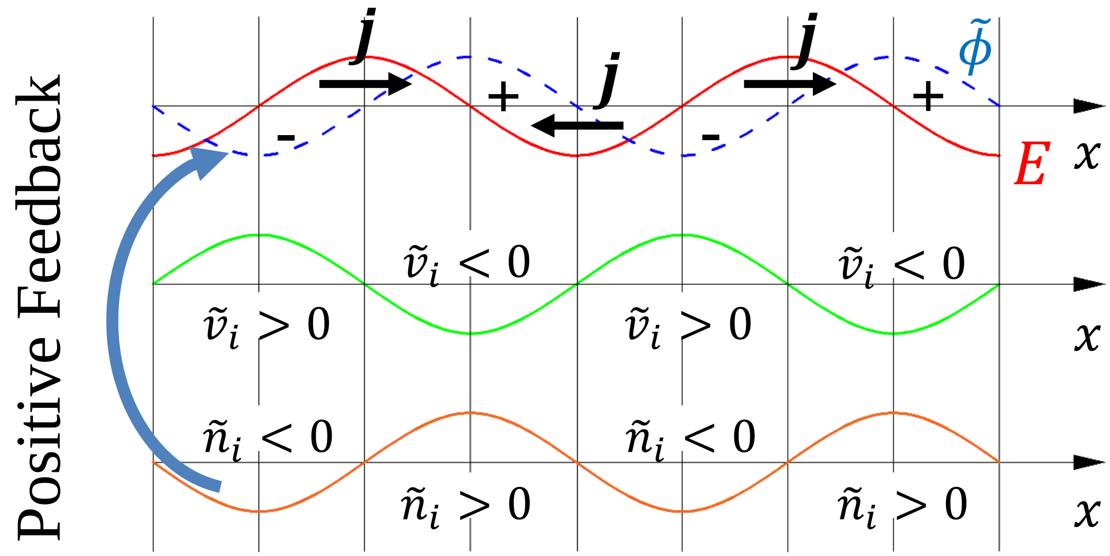
Superposition of small scale 2D modes and large scale axial mode

Axial-flow instability and large scale structures



Inherent instability of the current flow: Dissipative response of the electron flow + ballistic (inertial) response of ions

Electron current: collisional and/or anomalous



$$J_e = \sigma_{a,c} E$$

$$\frac{m_i v^2}{2} + e\phi = const$$

$$\tilde{v} = -\frac{e\tilde{\phi}}{m_i v_{i0}}$$

$$nv = const$$

$$\tilde{n} = -\frac{\tilde{v}_i n_0}{v_{i0}} = -\frac{en\tilde{\phi}}{m_i v_{i0}}$$

Koshkarov et al, Phys Plasmas 2017

Ion response is inertial: in phase with potential

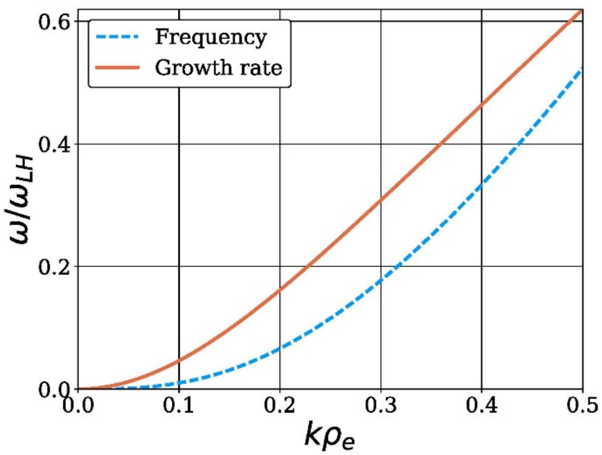
Resistive current flow instabilities

- No density gradient required
- Occur in the direction of the current flow
- Either along the stationary ExB flow or along the ion flow

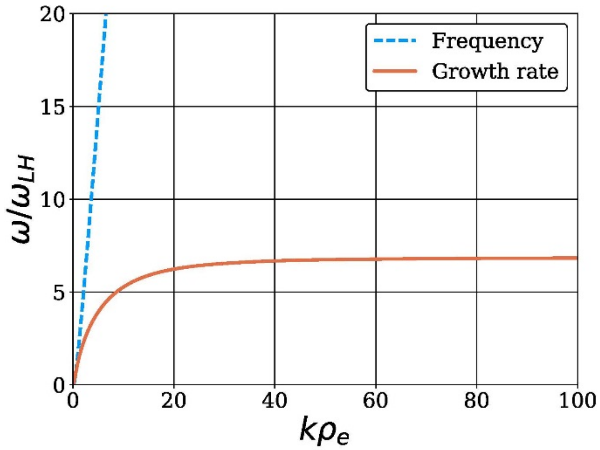
Fish, Litvak 2001
Chable, Rogier, 2005
Fernandez et al 2008
Koshkarov et al, 2017, 2018

In case of the are axial current (ion) flow is an important ingredient of breathing mode oscillations? Chable, Rogier 2005

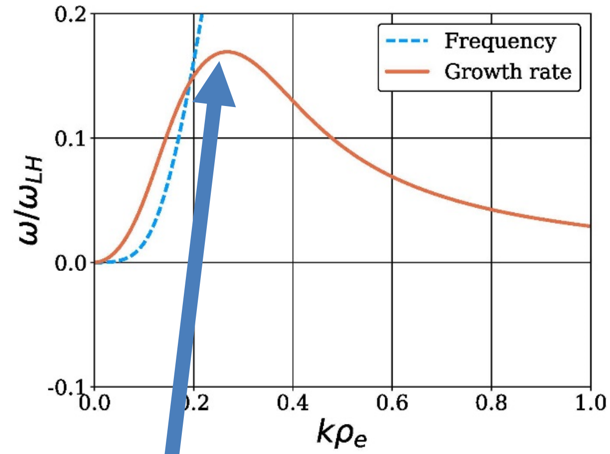
Electron inertia selects the most unstable mode



mobility



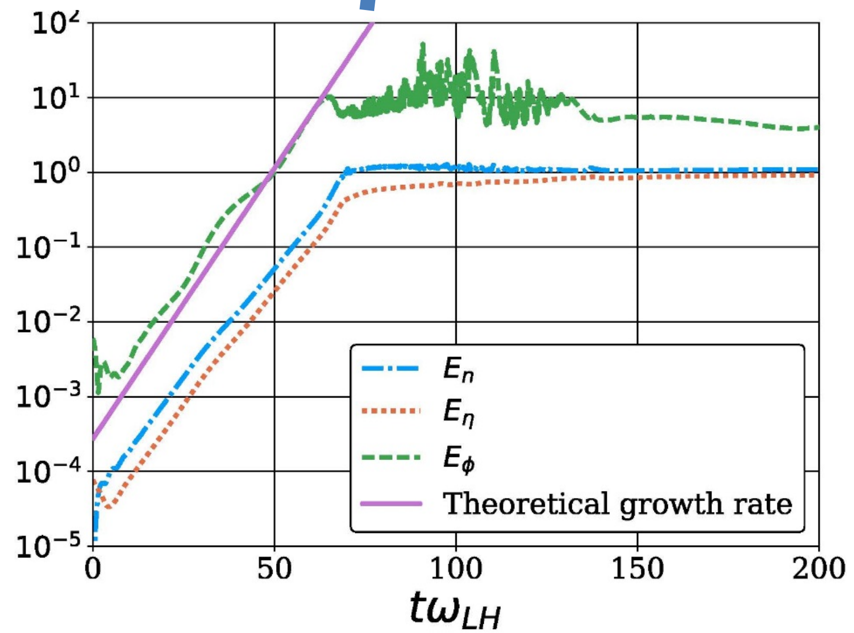
mobility +diffusion



+ inertia

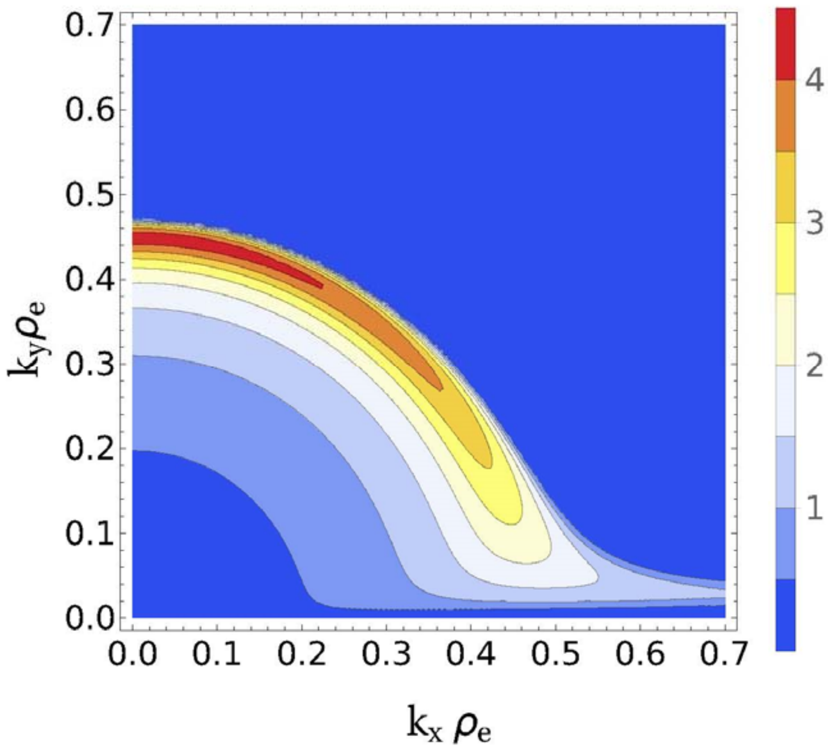
Linear growth and nonlinear saturation

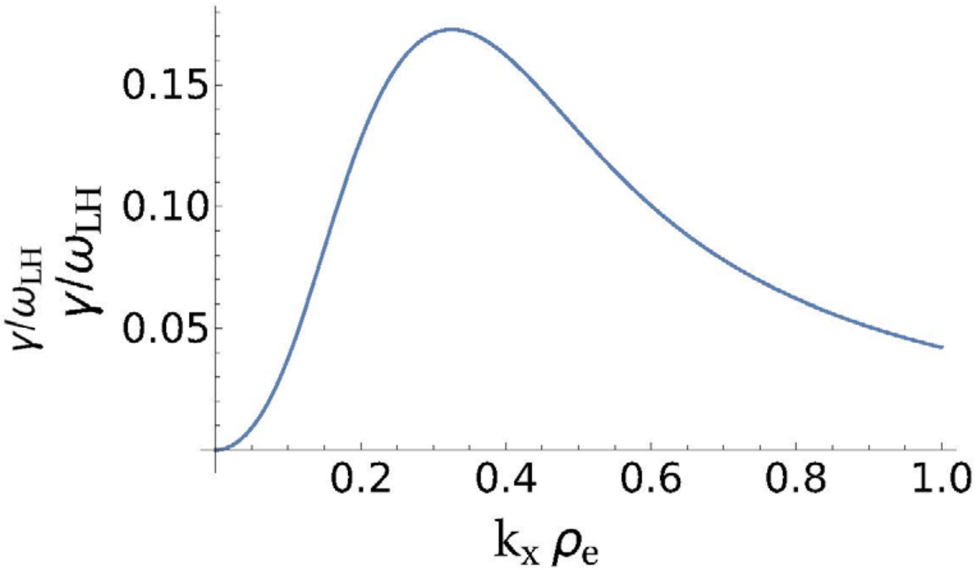
Koshkarov et al, Physics of Plasma 2017



Linear instabilities

$$\frac{c_s^2 k^2}{(\omega - v_0 k_x)^2} = \frac{v_d k_y + \rho_e^2 k^2(\omega - u_0 k_y + i\nu)}{\omega - u_0 k_y + \rho_e^2 k^2(\omega - u_0 k_y + i\nu)},$$





Axial mode instability, slow but hard to stabilize (ions)